The Simple Behaviour of Complex Systems Explained?

REVIEW
M. STREVENS
Bigger Than Chaos: Understanding Complexity Through Probability
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1 Introduction

This book aims to explain, by appealing to the mathematical method of arbitrary functions (MAF) initiated by Hopf and Poincaré, how the many and various interactions of the parts of a complex system often result in simple probabilistic patterns of behaviour. A complex system is vaguely defined as a system of many parts (called *enions*) which are somewhat autonomous but strongly interacting; (italicised words are Strevens' jargon). Strevens says that a system shows *simple* behaviour when it can be described mathematically with a small number of variables.[1] A philosophical treatment of complex systems, the MAF, and the emergence of simple probabilistic patterns is welcome because these important topics have been rather neglected.

The book proceeds as follows: The Introduction (Chapter 1) is followed by a discussion in Chapters 2 and 3 of the MAF. This discussion is divided into an informal part, where the results are explained, and a formal part where the results are proven. Strevens' strategy here is to prove theorems from strong assumptions, and then to argue that they are approximately true if, more realistically, the assumptions are approximately fulfilled. In Chapter 4 these results are applied to explain the simplicity of complex systems. Finally, Chapter 5 contains very brief remarks on the philosophical implications for the higher-level sciences. No previous knowledge about complex systems or

[1] Hence systems exhibiting deterministic chaos can show simple behaviour.

the MAF is presupposed. Basic knowledge of probability theory suffices to understand the main arguments, and basic measure theory to master the formal parts.

Nevertheless, the discussion is interesting both to experienced readers and to novices. The book provides a good introduction to the MAF and contains some interesting original material. However, the book promises more than it delivers: There are no convincing examples, and one's ultimate conclusion is that we do not yet know whether the book's proposed explanation of the simple behaviour of complex systems succeeds. Furthermore, there are some conceptual lacunae and technical mistakes. Still, Strevens' arguments suggest at least a possible explanation of the simple behaviour of complex systems, and this is an important contribution given the difficulty of the task at hand. Let me go into the details. I will concentrate on the main chapters, Chapters 2–4, and, in order to advance the discussion, I will focus on my objections.

2 The Method of Arbitrary Functions

Strevens calls the scheme for explaining simple behaviour that he advocates *enion probability analysis* (EPA). It consists of three stages. First, a probability is assigned to events that a specific enion in a given microstate will next be in a given macrostate, the *(single microstate) enion probability*; where *microstates* describe all information about the enions and *macrostates* describe only enion statistics. Assume that the *probabilistic supercondition* holds, i.e. that these enion probabilities depend only on macrostates, and that the outcomes of any two different enion probabilities are probabilistically independent. Then, second, the enion probabilities can be aggregated, often by applying the law of large numbers, to yield probabilistic relations only between macrostates. Third, a law about macrostates is derived from these probabilities, which is simple because it makes no reference to microstates. Most of the book is concerned with showing, by appealing to the MAF, that complex systems satisfy the probabilistic supercondition.

Chapter 2 and the first part of Chapter 3 explain two core ideas of the MAF. Strevens' results here are a slight variation of classical results and are rather obvious once these results are known (see in particular Hopf [1934], [1936]; readers of this journal may also recall von Plato [1983]). His results differ because, Strevens argues, for the application to complex systems it is important not to consider limiting behaviour as the classical treatments do (e.g. a limit in which time or the number of sections in a roulette wheel goes to infinity). Let me explain the two core ideas in Strevens' terms.

First, consider a *probabilistic experiment*[2] C consisting of a (i) deterministic mechanism, such as the evolution function of a roulette wheel with 36 equally spaced red and black sections; (ii) the variables V which provide initial conditions for the mechanism, such as the initial angular velocity with which the roulette wheel is spun; (iii) a set of outcomes, such as 'landing red' and 'landing black' for the roulette wheel; and (iv) a density quantifying the frequency or probability of the possible values of V, such as the probability density of initial angular velocities imparted to the wheel by a croupier. Now assume that the experiment is *microconstant* relative to an outcome O, meaning that the space of possible values of V can be partitioned into micro-sized contiguous sets such that across the different sets, the proportion of values which lead to O is a constant, called the *strike ratio* C_O. Then for any density which is *macroperiodic* (meaning: constant on each set in the partition), the probability that the experiment yields O is C_O. For instance, the roulette wheel is microconstant relative to the outcome 'landing red' because the space of possible values of initial angular velocities can be partitioned into micro-sized contiguous sets such that across the different sets the proportion of velocities which lead to the outcome red is a constant, say one half. If the density representing the initial angular velocity of the croupier is constant on each set in the partition, then the probability of the experiment landing red will be one half (even though the value of the density can vary greatly from one set of the partition to another).

Strevens (pp. 60–1) argues that all mechanisms of microconstant experiments show sensitivity to initial conditions as usually understood in chaos theory. It is true that microconstancy implies that there is *some* ε which we regard as small such that for any initial condition v_1 there is an initial condition v_2 which is less than the distance ε apart from v_1 and which yields the opposite outcome. However, it is not true that microconstant experiments always show sensitivity to initial conditions as understood in chaos theory, namely that for *every* ε and every initial condition v_1 there is an initial condition v_2 which is less than ε apart from v_1 and which leads to a different outcome. For instance, imagine a roulette wheel whose position after t time units is $v + t$, where v represents the initial conditions, and where we choose

[2] Despite the anthropomorphic word 'experiment', this need not involve any human action or intention.

$v \in [0, 1]$ and assume that the wheel has a circumference of 1; hence positions which differ by a natural number are identified. Assume that the wheel is divided into 36 equally spaced red and black sections and that the mechanism is stopped after $\frac{34}{36}$ time units. Then the experiment is microconstant, but solutions starting close together stay close for all times;[3] hence very close initial conditions nearly always lead to the same outcome.

The second core idea of the MAF has to do with the probabilistic independence of two (or more) microconstant experiments. Given two causally isolated microconstant experiments C and D with strike ratios C_{O_1} and D_{O_2}, Strevens defines combined experiments in such away that the combined experiment is microconstant relative to the outcome O_1 and O_2 with strike ratio C_{O_1} and D_{O_2}. It follows that if the joint density is macroperiodic, the outcomes are probabilistically independent.

The second part of Chapter 3 contains some new and interesting results: most notably, that specific causal couplings of microconstant experiments yield probabilistically independent outcomes and that some experiments whose states are deterministically chained maintain macroperiodic distributions and hence yield probabilistically independent outcomes. These results hold under the additional assumptions that the transformation T representing the chaining mechanism or the causal coupling is *strongly inflationary*, i.e. T maps every subset of its domain to a larger set, and is *microlinear*, i.e. T is linear or affine once restricted to any set of some partition of its domain into contiguous micro-sized regions.

The main proofs in Chapters 2 and 3 are correct; but the discussion is sometimes strange; e.g., the term 'contiguous', which, as we have seen, figures in important definitions is never formally defined (pp. 128, p. 134, p. 220). What is more is that I have doubts about *Approximation 3.18.3* of the main theorem on deterministically chained experiments (p. 248). This approximation addresses the worry that when a transformation representing the chaining that is only approximately microlinear is repeatedly applied to a macroperiodic density, this may eventually result in a density which is not even approximately macroperiodic. Strevens argues that this worry is unfounded because strongly inflationary transformations will tend to stretch out any area of non-macroperiodicity. But this is not convincing. For instance, for the so-called exact systems, which include many strongly inflationary

[3] In other examples differences in initial conditions can even shrink for microconstant experiments.

transformations, any initial density will (sometimes quickly) converge to the invariant measure of the system. And many invariant measures are not approximately macroperiodic (cf. Berger [2001], Chapter 4). This is problem is potentially serious because this approximation plays a crucial role in the application to the complex systems in Chapter 4.

3 Complex Systems

In the first part of Chapter 4 Strevens states general conditions under which, he argues, the simple behaviour of complex systems can be explained. The evolution of complex systems is represented by sequences of microstates. Given a microstate, the outcomes of many *microdynamic probabilistic experiments* determine the next microstate. The formal results of Chapters 2 and 3 are applied so as to conclude that the outcomes of these microdynamic probabilistic experiments are probabilistically independent. Strevens argues that this implies that every (single microstate) enion probability depends only on macrostates and that the outcomes of any two different enion probabilities are probabilistically independent. Thus the probabilistic supercondition is satisfied, and EPA can be applied to obtain laws about macrostates.

Here I should note that the title *'Bigger Than Chaos'* and the discussion about chaos is confusing because two different meanings of chaos (enions interacting in many and various ways, and deterministic chaos as in chaos theory) are conflated (p. 332). Deterministic chaos is not a main concern of the book, although some results require strong inflation, which is sometimes found, but neither necessary nor sufficient for deterministic chaos (Werndl [2009a], pp. 209–211). What the title expresses is the main theme, viz. that probabilities of microconstant experiments are indifferent to the detailed complicated micro-dynamics of complex systems and create simple behaviour.

The general argument for the simplicity of complex systems simply assumes that microdynamic probabilistic experiments are microconstant and have macroperiodic densities (pp. 278, p. 283). And it requires that the coupling and the chaining transformations are strongly inflationary. All will agree that in the final analysis this has to be verified for each complex system. But we should note that Strevens' remark that strong inflation 'is in fact not such a rare thing' (p. 279) is controversial since this is a very strong condition. For instance, although several very simple models of deterministic chaos are strongly inflationary, many chaotic systems such as the logistic map or the Lorenz system are not; and it is conjectured that more realistic chaotic models are typically not strongly inflationary (Smith et al. [1999], pp. 2861–2).

Furthermore, the crucial formal conclusion that the (single microstate) enion probabilities only depend on macrostates is established only informally (Section 4.4), leaving uncertainty whether the conclusion is true. Given that the main results in Chapters 2 and 3 were really proven, one would have expected proper formal results here also.

Moreover, important paradigms of complex systems are what is often called self-organised systems, where enions interact much more strongly with their neighbours than with other enions (Camezine et al. [2001]). As Strevens points out (p. 291), EPA alone cannot explain the simplicity of these systems. Here additionally other techniques, such as the renormalisation group, are needed.

4 Examples: Statistical Physics, Population Ecology and the Social Sciences

In the second part of Chapter 4 EPA is applied to concrete examples. In statistical physics Strevens claims that, by considering the collision of particles in hard-sphere systems, the application of EPA yields the Maxwell-Boltzmann distribution, or at least the assumptions such as 'molecular chaos' needed to derive this distribution. And he spectulates that the same considerations can be used to provide an understanding of all the dynamics properties of gases, such as the tendency to equilibrium. These ideas are interesting and worthy of further consideration.

But Strevens' discussion is rather disappointing: Several formal conclusions are only established informally, leaving uncertainty whether they are true. What is more is that there are several different derivations of the Maxwell-Boltzmann distribution, which depend on different assumptions (Uffink [2006], Sections 3 and 4; Uhlenbeck and Ford [1963]). Strevens neither states which of these sets of assumptions he wants to derive nor does he directly derive the Maxwell-Boltzmann distribution. And the exact assumptions needed for the known derivations of the Maxwell-Boltzmann distribution seem different from the ones Strevens obtains.

Furthermore, his comments about other approaches in statistical physics are sometimes erroneous. Let me give two examples. First, Strevens claims that 'the ergodic approach looks for a more extreme form of independence than I do' (p. 316). It is unclear what 'the ergodic approach' is because there are many different approaches to statistical physics employing ergodic theory. Strevens seems to think that 'the ergodic approach' requires systems to be Bernoulli systems, whilst he only requires that the evolution can be viewed as an irreducible and aperiodic Markov process. The discussion is quite informal; but if he really regarded the evolution as a discrete-time or continuous-time finite-state irreducible and aperiodic Markov process, this would actually *imply* that the system is Bernoulli (Ornstein and Weiss [1991], p. 22; Werndl [2009b]). Furthermore, several ergodic approaches impose conditions on the dynamics which are weaker than being Bernoulli, for instance, that the systems are ergodic or nearly ergodic (Frigg [2008], Sections 3.2.4 and 3.3.3; Uffink [2006], Sections 3–6). Ergodicity does not even imply any sensitivity to initial conditions. All this leads one to strongly doubt that his approach requires less independence than 'the ergodic approach'.

Second, Strevens claims that ergodic theory is unsuitable for treating complex systems because even the most general theorems in ergodic theory assume that the energy is conserved and that the evolution function is continuous (pp. 26–27). This is not so: Ergodic theory has been applied to several systems which do not conserve energy, e.g. the Lorenz system, and to many non-continuous evolution functions, e.g. the baker's transformation (Ornstein and Weiss [1991]). Generally, many ergodic theorems only require a measurable evolution function, including functions which are *nowhere* continuous, e.g. the Dirichlet function (Berger [2001], Chapter 3).

By appealing to EPA, Strevens suggests some interesting possible explanations of the simple behaviour in population ecology and in the social sciences. But there are no concrete models, and he admits that not enough is known to really say whether EPA applies (pp. 327, p. 355). So in the end we can conclude only that still a lot of work is needed to explain the simple behaviour of complex systems; exciting work for the future!

Despite these problems, the book will be valuable because of its good introduction to the MAF, its new results concerning the MAF, and its stimulating ideas about how to explain the simple behaviour of complex systems.

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